

## STEP MATHEMATICS

### Mapping document for the new 2019 specifications

This document indicates where the various sections of the old STEP I and II specification and STEP III specification have mapped to in the new specifications.

This document should be read alongside the STEP Specifications for June 2019 which can be found on the Cambridge Assessment Admissions Testing website <https://www.admissionstesting.org/>.

This document has been created as part of the STEP Support Programme <https://maths.org/step/>.

#### Key:

This colour indicates that the material came from the old STEP I and II specification

This colour indicates that the material came from the old STEP III specification

Highlighted in yellow indicates that this material is new to the immediately previous specification (but the material may have been present in older versions)

There have been some (very small) new additions for 2020 – these are written in UPPER CASE AND UNDERLINED. These are on pages 3, 12 and 14 of this document.

# MATHEMATICS 1

## Section A: Pure Mathematics

### Content

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#### Proof

##### (Old STEP I/II)

(Note – proof by induction is no longer restricted to “simple cases”)

Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including proof by deduction, proof by exhaustion, **proof by induction**.

**Understand and use the terms ‘necessary and sufficient’ and ‘if and only if’.**

Disproof by counter-example.

Proof by contradiction (including proof of the irrationality of  $\sqrt{2}$  and the infinity of primes, and application to unfamiliar proofs).

#### Algebra and functions

##### (Old STEP I/II)

**Know**, understand and use the laws of indices for all rational exponents.

Use and manipulate surds, including rationalising the denominator.

Work with quadratic functions and their graphs; the discriminant of a quadratic function, including the conditions for real and repeated roots; completing the square; solution of quadratic equations including solving quadratic equations in a function of the unknown.

Solve simultaneous equations in two (**or more**) variables by elimination and by substitution; including, **for example**, one linear and one quadratic equation.

Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions.

Express solutions through correct use of ‘and’ and ‘or’, or through set notation.

Represent linear and quadratic inequalities such as  $y > x + 1$  and  $y > ax^2 + bx + c$  graphically.

**Solve inequalities and interpret them graphically; including, but not limited to, those involving rational algebraic**

**expressions (e.g.,  $\frac{1}{a-x} > \frac{x}{x-b}$ ), trigonometric functions,**

**exponential functions, and the modulus function.**

Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation, and simple algebraic division; use of the factor theorem **and the remainder theorem; use of equating coefficients in identities.**

Note that understanding the relationships between cubic and higher order equations is now in STEP II

**Know, understand and use the relationship between the roots and coefficients of quadratic equations.**

Simplify rational expressions including by factorising and cancelling, and algebraic division (by linear **and higher degree** expressions).

Understand and use graphs of functions; sketch curves defined by simple equations including polynomials, the modulus of linear

**and other** functions,  $y = \frac{a}{x}$  and  $y = \frac{a}{x^2}$  **and other rational**

**functions such as**  $y = \frac{x}{(x-a)^2}$  (including their vertical and

horizontal asymptotes); **behaviour as**  $x \rightarrow \pm\infty$ ; interpret the algebraic solution of equations graphically; use intersection points of graphs to solve equations.

Understand and use proportional relationships and their graphs.

**NEW ADDITION HERE FOR 2020**

Understand and use THE DEFINITION OF A FUNCTION; DOMAIN AND RANGE OF FUNCTIONS; composite functions; inverse functions and their graphs.

Understand the effect of simple transformations on the graph of  $y = f(x)$  including sketching associated graphs:

$y = af(x)$ ,  $y = f(x) + a$ ,  $y = f(x + a)$ ,  $y = f(ax)$ , and combinations of these transformations.

Note – quadratic denominators which are not squared linear terms are now in STEP III

Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than three terms, numerators constant or linear).

**Understand what is meant by the limit of a function  $f(x)$  as  $x$  tends to a specific value at which the function is undefined, including the notation  $x \rightarrow \infty$ , and be able to find such limits in simple cases.**

Use functions in modelling, including consideration of limitations and refinements of the models.

**Coordinate geometry  
in the  $(x, y)$  plane  
(Old STEP I/II)**

These circle theorems were always assumed knowledge in STEP I and II, but not previously explicitly mentioned.

**Know**, understand and use the equation of a straight line, including the forms  $y - y_1 = m(x - x_1)$  and  $ax + by + c = 0$ ; gradient conditions for two straight lines to be parallel or perpendicular.

Be able to use straight line models in a variety of contexts.

**Know**, understand and use the coordinate geometry of the circle including using the equation of a circle in the form  $(x - a)^2 + (y - b)^2 = r^2$ ; completing the square to find the centre and radius of a circle; **know, understand** and use basic circle theorems:

- ***The angle subtended by an arc at the centre is twice the angle it subtends at the circumference.***
- The angle on the circumference subtended by a diameter is a right angle.
- ***Two angles subtended by a chord in the same segment are equal.***
- A radius or diameter bisects a chord if and only if it is perpendicular to the chord.
- For a point P on the circumference, the radius or diameter through P is perpendicular to the tangent at P.
- ***For a point P on the circumference, the angle between the tangent and a chord through P equals the angle subtended by the chord in the alternate segment.***
- ***Opposite angles of a cyclic quadrilateral are supplementary.***

Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms.

Use parametric equations in modelling in a variety of contexts.

## Sequences and series

### (Old STEP I/II)

**Know**, understand and use the binomial expansion of  $(a+bx)^n$  for positive integer  $n$ ; the notations  $n!$  and  $nCr$  (**and**  $\binom{n}{r}$  **and**  ${}^nC_r$ ) **and their algebraic definitions**; link to binomial probabilities.

Extend the binomial expansion of  $(a+bx)^n$  to any rational  $n$ , including its use for approximation; be aware that the expansion is valid (**converges**) for  $\left|\frac{bx}{a}\right| < 1$  (proof not required).

**Use**  $n!$  **and**  ${}^nC_r$  **in the context of permutations and combinations.**

Work with sequences including those given by a formula for the  $n^{\text{th}}$  term and those generated by a simple relations of the form  $x_{n+1} = f(x_n)$ , **or**  $x_{n+1} = f(x_n, x_{n-1})$ ; increasing sequences; decreasing sequences; periodic sequences.

Understand and use sigma notation for sums of series.

Understand and work with arithmetic sequences and series, including **knowledge of** the formulae for  $n^{\text{th}}$  term and the sum to  $n$  terms.

Understand and work with geometric sequences and series including **knowledge of** the formulae for the  $n^{\text{th}}$  term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of  $|r| < 1$ .

**Understand what is meant by the limit of a sequence, including the notation**  $x_n \rightarrow a$  **as**  $n \rightarrow \infty$ , **and be able to find such a limit in simple cases.**

Use sequences and series in modelling.

## Trigonometry

### (Old STEP I/II)

**Know**, understand and use the definitions of sine, cosine, and tangent for all arguments; the sine and cosine rules; the area of a triangle in the form  $\frac{1}{2}ab\sin C$ .

Work with radian measure, including use for arc length and area of sector.

**Know**, understand and use the standard small angle approximations of  $\sin \theta$ ,  $\cos \theta$ , and  $\tan \theta$ :

$$\sin \theta \approx \theta, \cos \theta \approx 1 - \frac{\theta^2}{2}, \tan \theta \approx \theta \text{ where } \theta \text{ is in radians.}$$

Understand and use the sine, cosine, and tangent functions; their graphs, symmetries, and periodicity.

Know and use exact values of  $\sin \theta$  and  $\cos \theta$  for

$$\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi \text{ and integer multiples.}$$

Know and use exact values of  $\tan \theta$  for

$$\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \pi \text{ and (appropriate) integer multiples.}$$

**Know**, understand and use the definitions of  $\sec$ ,  $\operatorname{cosec}$ , and  $\cot$  and of  $\sin^{-1}$ ,  $\cos^{-1}$ , and  $\tan^{-1}$ ; their relationships to  $\sin$ ,  $\cos$  and  $\tan$ ; understand their graphs, their ranges and domains.

**Know**, understand and use  $\tan \theta = \frac{\sin \theta}{\cos \theta}$ .

**Know**, understand and use  $\sin^2 \theta + \cos^2 \theta = 1$ ,

$$\sec^2 \theta = 1 + \tan^2 \theta, \text{ and } \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta.$$

**Know**, understand and use double angle formulae; use of formulae for  $\sin(A \pm B)$ ,  $\cos(A \pm B)$ , and  $\tan(A \pm B)$ ; understand geometrical proofs of these formulae.

Understand and use expressions for  $a \cos \theta + b \sin \theta$  in the equivalent forms of  $r \cos(\theta \pm \alpha)$  or  $r \sin(\theta \pm \alpha)$ .

**Find general solutions to trigonometric equations, including quadratic equations in  $\sin$ ,  $\cos$ , or  $\tan$  and equations involving linear multiples of the unknown angle; for**

**example**,  $\sin\left(3x + \frac{\pi}{5}\right) = \frac{1}{2}$ .

Construct proofs involving trigonometric functions and identities.

Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.

## Exponentials and logarithms (Old STEP I/II)

Note that the exponential series  $e^x = 1 + x + \dots + \frac{x^n}{n!} + \dots$  is now in STEP II

Know and use the function  $a^x$  and its graph, where  $a$  is positive.

Know and use the function  $e^x$  and its graph.

Know that the gradient of  $e^{kx}$  is equal to  $ke^{kx}$ , and hence understand why the exponential model is suitable in many applications.

Know and use the definition of  $\log_a x$  as the inverse of  $a^x$ , where  $a$  is positive ( $a \neq 1$ ) and  $x > 0$ .

Know and use the function  $\ln x$  and its graph.

Know and use  $\ln x$  as the inverse function of  $e^x$ .

**Know**, understand and use the laws of logarithms:

$$\log_a x + \log_a y = \log_a xy ;$$

$$\log_a x - \log_a y = \log_a \frac{x}{y} ;$$

$$k \log_a x = \log_a x^k$$

(including, for example,  $k = -1$  and  $k = \frac{-1}{2}$ ).

**Understand and use the change of base formula for logarithms:**

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Solve equations of the form  $a^x = b$ .

Use logarithmic graphs to estimate parameters in relationships of the form  $y = ax^n$  and  $y = kb^x$ , given data for  $x$  and  $y$ .

Understand and use exponential growth and decay; use in modelling (examples may include the use of  $e$  in continuous compound interest, radioactive decay, drug concentration decay, or exponential growth as a model for population growth); consideration of limitations and refinements of exponential models.

## Differentiation

### (Old STEP I/II)

**Candidates should have an informal understanding of continuity and differentiability.**

Understand and use the derivative of  $f(x)$  as the gradient of the tangent to the graph of  $y=f(x)$  at a general point  $(x,y)$ ; the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second **and higher derivatives**; differentiation from first principles for small positive integer powers of  $x$ , and for  $\sin x$  and  $\cos x$ .

Understand and use the second derivative as the rate of change of gradient; connection to convex and concave sections of curves and points of inflection.

Differentiate  $x^n$ , for rational values of  $n$ , and related constant multiples, sums and differences.

Differentiate  $e^{kx}$ ,  $a^{kx}$ ,  $\sin kx$ ,  $\cos kx$ ,  $\tan kx$  **and other trigonometric functions** and related sums, differences and constant multiples.

**Know**, understand and use the derivative of  $\ln x$ .

Apply differentiation to find gradients, tangents and normals, maxima and minima and stationary points, points of inflection.

Identify where functions are increasing or decreasing.

Differentiate using the product rule, the quotient rule, and the chain rule, including problems involving connected rates of change and inverse functions.

Differentiate simple functions and relations defined implicitly or parametrically, for first **and higher** derivatives.

**Apply the above to curve sketching.**

Construct simple differential equations in pure mathematics and in context (contexts may include kinematics, population growth, and modelling the relationship between price and demand).



## Integration

(Old STEP I/II)

**Candidates should have an informal understanding of integrability.**

Know and use the Fundamental Theorem of Calculus, **including applications to integration by inspection.**

Note that volumes of revolution are now in STEP III

Integrate  $x^n$  (**including**  $n = -1$ ), and related sums, differences and constant multiples.

Integrate  $e^{kx}$ ,  $\sin kx$ , and  $\cos kx$ , and related sums, differences, and constant multiples.

Note that improper integrals are now in STEP II

Evaluate definite integrals; use a definite integral to find the area under a curve and the area between two curves.

Understand and use integration as the limit of a sum.

Carry out simple **and more complex** cases of integration by substitution and integration by parts; understand these methods as the inverse processes of the chain and product rules respectively.

(Integration by substitution includes finding a suitable substitution and is **not** limited to cases where one substitution will lead to a function which can be integrated; integration by parts includes more than one application of the method but excludes reduction formulae.)

Integrate using partial fractions that are linear **and repeated linear** in the denominator.

Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions.

(Separation of variables may require factorisation involving a common factor.)

Interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution; includes links to kinematics.

## Numerical methods

### (New addition to STEP)

Locate roots of  $f(x)=0$  by considering changes of sign of  $f(x)$  in an interval of  $x$  on which  $f(x)$  is sufficiently well-behaved.

Understand how change of sign methods can fail.

Solve equations approximately using simple iterative methods; be able to draw associated cobweb and staircase diagrams.

Solve equations using the Newton-Raphson method and other recurrence relations of the form  $x_{n+1}=g(x_n)$ .

Understand how such methods can fail.

Understand and use numerical integration of functions; including the use of the trapezium rule, and estimating the approximate area under a curve and limits that it must lie between.

Use numerical methods to solve problems in context.

## Vectors

### (Old STEP I/II)

Note that Vector equations of lines and the scalar product are now in STEP II

Use vectors in two dimensions and in three dimensions.

Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form.

Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.

Understand and use position vectors; calculate the distance between two points represented by position vectors.

***Know how to relate the position vector of the point that divides AB in a given ratio to the position vectors of the points A and B.***

Use vectors to solve problems in pure mathematics and in context, including forces and kinematics.

## Section B: Mechanics and Probability/Statistics

### Content

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#### Quantities and units in mechanics (Old STEP I/II)

**Know**, understand and use fundamental quantities and units in the S.I. system: length, time, mass.

**Know**, understand and use derived quantities and units: velocity, acceleration, force, weight, moment.

#### Kinematics

##### (Old STEP I/II)

Note that Energy and Collisions are now in STEP II

**Know**, understand and use the language of kinematics: position, displacement, distance travelled, velocity, speed, acceleration.

Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time, and interpretation of gradient; velocity against time, and interpretation of gradient and area under the graph.

**Know**, understand, use and derive the formulae for constant acceleration for motion in a straight line; extend to 2 dimensions using vectors.

Use calculus in kinematics for motion in a straight line:

$$v = \frac{dr}{dt}, a = \frac{dv}{dt} = \frac{d^2r}{dt^2}, r = \int v dt, v = \int a dt ; \text{ extend to 2}$$

dimensions using vectors.

Model motion under gravity in a vertical plane using vectors; projectiles.

## Forces and Newton's laws

### (Old STEP I/II)

Understand the concept of a force; understand and use Newton's first law.

**Know**, understand and use Newton's second law for motion in a straight line, including situations where forces need to be resolved (in 2 **or** 3 dimensions); application to problems involving smooth pulleys and connected particles.

Understand and use weight, and motion in a straight line under gravity; gravitational acceleration,  $g$ , and its value in S.I. units to varying degrees of accuracy.

(The inverse square law for gravitation is not required and  $g$  may be assumed to be constant, but students should be aware that  $g$  is not a universal constant but depends on location.)

**Know**, understand and use Newton's third law; resolving forces in 2 **or** 3 dimensions; equilibrium of a particle under forces in 2 **or** 3 dimensions.

### NEW ADDITION HERE FOR 2020

Understand and use addition of forces; resultant forces; THE TRIANGLE OF FORCES FOR EQUILIBRIUM PROBLEMS; dynamics for motion in a plane.

**Know**, understand and use the  $F \leq \mu R$  model for friction; coefficient of friction; motion of a body on a rough surface; limiting friction and statics.

## Moments

### (Old STEP I/II)

Understand and use moments in simple static contexts; **questions will not be restricted to those involving forces in two perpendicular directions. Questions may be set on equilibria of rigid bodies, including bodies in contact, and on breaking of equilibrium, for example by toppling or slipping.**

**Understand and use the idea of centre of mass; the position of the centre of mass of any shapes used will either be given or deducible by the use of symmetry.**

## Statistical sampling

Was previously “Assumed knowledge”

Understand and use the terms ‘population’, ‘sample’ and ‘**random sample**’.

## Data presentation and interpretation

Was previously “Assumed knowledge”

Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency, and connect to probability distributions.

Interpret measures of central tendency and variation, extending to standard deviation.

Be able to calculate standard deviation, including from summary statistics.

## Probability (Old STEP I/II)

Note that these two formulae were not explicitly required before 2019

Understand and use mutually exclusive, independent, **and complementary** events when calculating probabilities. Link to discrete and continuous distributions.

Understand and use conditional probability, including the use of tree diagrams, Venn diagrams, two-way tables.

**Know, understand and use the formula:**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

**Know**, understand and use the conditional probability formula:

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Modelling with probability, including critiquing assumptions made and the likely effect of more realistic assumptions.

**Use combinatorial arguments, including the use of  $n!$  and  ${}^n C_r$ , in the context of calculating probabilities.**

## Statistical distributions (Old STEP I/II)

### NEW ADDITION HERE FOR 2020

Note that the Poisson distribution and general continuous probability distributions are now in STEP II

### UNDERSTAND INFORMALLY THE CONCEPT OF A RANDOM VARIABLE

Understand and use simple, discrete probability distributions (calculation of expectation and variance of discrete random variables is **included**), including the Binomial distribution as a model; calculate probabilities using the Binomial distribution.

***The discrete uniform distribution as a model; calculate probabilities using the discrete uniform distribution.***

Understand and use the Normal distribution; find probabilities using the Normal distribution; ***convert to the standard Normal distribution by translation and scaling.***

Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the Binomial or Normal model may not be appropriate.

## Statistical hypothesis Testing (Old STEP I/II)

Understand and apply the language of statistical hypothesis testing, developed through a Binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tailed test, 2-tailed test, critical value, critical region, acceptance region, ***p***-value.

Conduct a statistical hypothesis test for the proportion in the Binomial distribution and interpret the results in context.

Understand that a sample is being used to make an inference about the population, and appreciate how the significance level and the probability of incorrectly rejecting the null hypothesis are related.

Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given, or assumed variance and interpret the results in context.

## MATHEMATICS 2

The specification for Mathematics 2 assumes full knowledge and understanding of the relevant parts of the specification for Mathematics 1 as set out in the introduction. Candidates should be aware that questions in this paper may be set on any relevant parts of the Mathematics 1 or Mathematics 2 specifications.

### Section A: Pure Mathematics

#### Content

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#### Complex numbers

##### (Old STEP III)

Solve any quadratic equation with real **or complex** coefficients; solve cubic or quartic equations with real **or complex** coefficients (given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics).

Add, subtract, multiply, and divide complex numbers in the form  $x+iy$  with  $x$  and  $y$  real; understand and use the terms 'real part' and 'imaginary part'.

Understand and use the complex conjugate; know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs.

Use and interpret Argand diagrams.

Convert between the Cartesian form and the modulus-argument form of a complex number (knowledge of radians is assumed).

Multiply and divide complex numbers in modulus-argument form (knowledge of radians and compound angle formulae is assumed).

Construct and interpret simple loci in the Argand diagram such as, **but not limited to**,  $|z - a| = r$  and  $\arg(z - a) = \theta$  (knowledge of radians is assumed).

## Matrices

### (New addition to STEP)

Add, subtract, and multiply conformable matrices; multiply a matrix by a scalar.

Understand and use zero and identity matrices.

Use matrices to represent linear transformations in 2-D; successive transformations; single transformations in 3-D (3-D transformations confined to reflection in one of  $x=0, y=0, z=0$  or rotation about one of the coordinate axes). (Knowledge of 3-D vectors is assumed.)

Find invariant points and lines for a linear transformation.

Calculate determinants of  $2 \times 2$  matrices **and interpret as scale factors, including the effect on orientation.**

Understand and use singular and non-singular matrices; properties of inverse matrices.

Calculate and use the inverse of a non-singular  $2 \times 2$  matrix.

## Further algebra and functions

Old STEP I/II

Understand and use the relationship between roots and coefficients of polynomial equations up to quartic **and higher degree** equations.

Old STEP III

Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree).

Old STEP I/II

**Know and use partial fractions in which the denominator may include quadratic factors of the form  $a x^2 + c$  for  $c > 0$ , and in which the degree of the numerator may be equal to, or exceed, the degree of the denominator.**

Old STEP III

**Understand and use the method of differences for summation of series, including the use of partial fractions.**

Old STEP I/II

**Recognise and use the series expansion of  $e^x$ .**

Old STEP III

**Sketch curves of the form  $\frac{x^2}{a^2} \pm \frac{y^2}{b^2} = 1$ ; find equations of their asymptotes where appropriate.**



## Further calculus

Old STEP III

*Differentiate inverse trigonometric functions.*

Old STEP I/II

*Evaluate improper integrals where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinity.*

Old STEP III

*Integrate functions of the form  $(1+x^2)^{-1}$  and  $(1-x^2)^{-\frac{1}{2}}$  and be able to choose trigonometric substitutions to integrate associated functions.*

Old STEP I/II

*Integrate using partial fractions (including those with quadratic factors in the denominator).*

*Integrate using reduction formulae.*

## Further vectors (Old STEP I/II)

*Understand and use the vector and Cartesian forms of an equation of a straight line in 3-D.*

*Understand and use the scalar product of two vectors, including geometrical interpretation and formal algebraic manipulation; for example,  $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$*

## Section B: Mechanics

### Content

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#### Energy, work, and power

(Old STEP I/II)

*Understand and use the concepts of energy (kinetic and potential), work, and power.*

*Understand and use the principle of conservation of energy.*

#### Collisions

(Old STEP I/II)

*Understand the mechanics of collisions in simple situations.*

*Understand and use the principle of conservation of momentum and, when appropriate, the conservation of energy applied to collisions.*

*Understand and use the coefficient of restitution ( $e$ ) for collisions, including the special cases  $e = 1$  and  $e = 0$ .*

*Questions involving successive impacts may be set.*

*Knowledge of oblique impacts will not be required.*

#### Hooke's law

(Old STEP III)

*Know, understand and use Hooke's law for strings and springs, including the formula  $T = kx = \frac{\lambda x}{l}$  where  $k$  is the stiffness and  $\lambda$  is the modulus of elasticity.*

*Understand and use elastic potential energy, including*

*knowledge of the formula  $E = \frac{1}{2} k x^2 = \frac{\lambda x^2}{2l}$ .*

## Section C: Probability/Statistics

### Content

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#### Probability distributions

(Old STEP I/II)

*Know, understand and use the Poisson distribution; find probabilities using the Poisson distribution.*

*Calculate the mean and variance of the Poisson distribution.*

*Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the Binomial, the Normal or the Poisson model may not be appropriate.*

*Use the Poisson distribution as an approximation to the Binomial distribution and know under what conditions this is appropriate.*

*Use the Normal distribution as an approximation to the Binomial distribution or the Poisson distribution and know under what conditions these are appropriate.*

*Know, understand and use the continuous uniform distribution; find probabilities using the continuous uniform distribution.*

*Calculate the mean and variance of the continuous uniform distribution.*

*Understand and use the mathematics of continuous probability density functions and cumulative distribution functions; including finding probabilities and the calculation of mean, variance, median, mode, and expectation by explicit integration for a given (possibly unfamiliar) distribution; the notation  $f(x)=F'(x)$ .*



## MATHEMATICS 3

The specification for Mathematics 3 assumes full knowledge and understanding of the relevant parts of the specification for Mathematics 1 and Mathematics 2 as set out in the introduction. Candidates should be aware that questions in this paper may be set on any relevant parts of the Mathematics 1, Mathematics 2 or Mathematics 3 specifications.

### Section A: Pure Mathematics

#### Content

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#### Further Complex numbers

##### (Old STEP III)

**Know** and understand de Moivre's theorem and use it to find multiple angle formulae and sums of series.

Know and use the definition  $e^{i\theta} = \cos \theta + i \sin \theta$  and the form  $z = r e^{i\theta}$

Find the  $n$  distinct  $n^{\text{th}}$  roots of  $r e^{i\theta}$  for  $r \neq 0$  and know that they form the vertices of a regular  $n$ -gon in the Argand diagram.

Use complex numbers, including complex roots of unity, to solve geometric problems.

#### Further Matrices

##### (New addition to STEP)

Calculate determinants of  $3 \times 3$  matrices and interpret as scale factors, including the effect on orientation.

Calculate and use the inverse of non-singular  $3 \times 3$  matrices.

Solve three linear simultaneous equations in three variables by use of the inverse matrix.

Interpret geometrically the solution and failure of solution of three simultaneous linear equations.

#### Further algebra and functions (Old STEP III)

Find the Maclaurin series of a function including the general term.

**Know** and use the Maclaurin series for  $e^x$ ,  $\ln(1+x)$ ,  $\sin x$ ,  $\cos x$ , and  $(1+x)^n$ , and be aware of the range of values of  $x$  for which they are valid (proof not required).

## Further calculus

(Old STEP III)

*Calculate lengths of curves in Cartesian or parameterised Cartesian coordinates.*

Old STEP I/II

Derive formulae for and calculate volumes of revolution.

(Old STEP III)

Understand and evaluate the mean value of a function.

## Further vectors

(New addition to STEP)

Understand and use the vector and Cartesian forms of the equation of a plane.

Calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two planes and the angle between a line and a plane.

Find the intersection of a line and a plane.

Calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane.

***Understand and use the vector product, including the geometrical interpretation; use vector products to calculate the area of a triangle or parallelogram and to determine whether vectors are parallel.***

***Understand and use the equation of a line in the form***

$$(r - a) \times b = c \cdot 0$$

## Polar coordinates

(Old STEP III)

Understand and use polar coordinates and be able to convert between polar and Cartesian coordinates.

***(It will be assumed that  $r \geq 0$ ; the range of  $\theta$  will be given if appropriate.)***

Sketch curves with  $r$  given as a function of  $\theta$ , including the use of trigonometric functions.

Find the area enclosed by a polar curve.

## Hyperbolic functions

### (Old STEP III)

**Know**, understand and use the definitions of hyperbolic functions  $\sinh x$ ,  $\cosh x$ ,  $\tanh x$ ,  $\operatorname{sech} x$ ,  $\operatorname{cosech} x$ ,  $\operatorname{coth} x$  including their domains and ranges, and be able to sketch their graphs.

**Know, understand and use standard formulae for algebraic relations between hyperbolic functions, such as**

$$\cosh^2 x - \sinh^2 x = 1.$$

Differentiate and integrate hyperbolic functions.

Understand and be able to use the definitions of the inverse hyperbolic functions and their domains and ranges.

Derive and use the logarithmic forms of the inverse hyperbolic functions.

Integrate functions of the form  $(x^2 + 1)^{-\frac{1}{2}}$  and  $(x^2 - 1)^{-\frac{1}{2}}$  and be able to choose substitutions to integrate associated functions.

## Differential equations

### (Old STEP III)

Find and use an integrating factor to solve differential equations of the form  $\frac{dy}{dx} + P(x)y = Q(x)$ ; recognise when it is appropriate to do so.

Find both general and particular solutions to differential equations, **including by methods that will be indicated if appropriate.**

Use differential equations in modelling in kinematics and in other contexts.

Solve differential equations of the form  $y'' + ay' + by = 0$ , where  $a$  and  $b$  are constants, by using the auxiliary equation.

**Know**, understand and use the form of the solution of the differential equations in cases when the discriminant of the auxiliary equation is positive, zero, or negative.

Solve differential equations of the form  $y'' + ay' + by = f(x)$  where  $a$  and  $b$  are constants, by solving the homogeneous case and adding a particular integral to the complementary function (in cases where  $f(x)$  is a polynomial, exponential, or trigonometric function).

Solve the equation for simple harmonic motion  $\ddot{x} = -\omega^2 x$  and relate the solution to the motion, **and understand the implications in physical situations.**

Model damped oscillations using second order differential equations and interpret their solutions.

Analyse models of situations with one independent variable and two dependent variables as a pair of coupled first order simultaneous equations and be able to solve them, for example predator-prey models.

**Use given substitutions to transform differential equations.**



**Further collisions  
(Old STEP I/II)**

*Understand and be able to use the concept of impulse.*

*Analyse collisions involving oblique impacts, including the use of the coefficient of restitution. Questions involving successive impacts may be set.*

**Centre of mass  
(Old STEP I/II)**

*Understand and be able to apply the principle that the effect of gravity is equivalent to a single force acting at the body's centre of mass.*

*Find the position of the centre of mass of a uniform rigid body using symmetry.*

*Determine the centre of mass of a system of particles or the centre of mass of a composite rigid body.*

*Use integration to determine the position of the centre of mass of a uniform lamina or a uniform solid of revolution.*

**Circular motion  
(Old STEP III)**

Note – the parallel and perpendicular axis theorems are no longer in STEP

*Know, understand and use the definitions of angular velocity, velocity, speed, and acceleration in relation to a particle moving in a circular path with constant speed; includes the use of both  $\omega$  and  $\dot{\theta}$ .*

*Know, understand and use the relationships  $v=r\omega$  and*

$$a = \frac{v^2}{r} = r\omega^2 = \dot{\omega}v \text{ for motion in a circle with constant speed.}$$

*Analyse motion with variable speed on an arc of a circle, including motion in a vertical circle.*

*Moments of inertia will not be examined.*

**Differential equations  
(Old STEP III)**

*Use differential equations to analyse models of particles moving under the action of variable forces, where forces will not necessarily be given as a function of time; including*

*recognising when it is appropriate to use  $a = v \frac{dv}{dx}$*

*Solve the equation for simple harmonic motion  $\ddot{x} = -\omega^2 x$  and understand applications to physical situations, including the approximate simple harmonic motion of a pendulum.*

## Section C: Probability/Statistics

### Content

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#### Independent random variables (Old STEP III)

*Understand and use the idea of independent random variables.*

#### Algebra of expectation (Old STEP III)

Note that generating functions and covariance are no longer in STEP

*Know, understand and use the algebra of expectation:*

$$E(aX + bY + c) = aE(X) + bE(Y) + c$$

$$\text{Var}(X) = E(X^2) - E(X)^2$$

$$\text{Var}(aX + b) = a^2 \text{Var}(X)$$

Note that the central limit theorem is no longer in STEP

*and for independent random variables:*

$$\text{Var}(aX \pm bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$$

*Knowledge of the relation*

$$\text{Var}(aX \pm bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) \pm 2ab \text{Cov}(X, Y)$$

*will not be required.*

*Use cumulative distribution functions to calculate the probability density function of a related random variable; for example,  $X^2$  from  $X$ .*

*Knowledge of generating functions will not be required.*